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ORIGINAL RESEARCH

EULER'S EQUATIONS OF RIGID BODY: ITS CHAOS CONTROL, TRACKING AND SYNCHRONIZATION

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Abstract:
Introduction: Chaos synchronization and control in dynamical systems are essential applications of chaos theory. Chaos control is sometimes needed to refine the behavior of a chaotic model and to remove unexpected performance of power electronics. Synchronization of chaos also has useful applications to biological, chemical, physical systems and secure communications. Lyapunov exponents is one of a number of effective ways to describe chaotic properties of non-linear systems. If one of the Lyapunov exponents is greater than zero, the system is chaotic, and if at least two of the Lyapunov exponents are positive, the system is hyper-chaotic. The greater the number of positive Lyapunov exponents, the higher the degree of instability in the system. The Eulers' equation of the rigid body has many physical applications, thus, the need to further work on its synchronization using the active control method, which has been adjudged to be efficient in experiencing the transient performances of the controllers designed. The control of the chaotic rigid body has been achieved before now, but we have extended it to achieve tracking to a desired function, which underlies its usefulness.
Aims: The aim of this paper is to achieve synchronization of two chaotic rigid body systems, to control its chaotic state and to track to a desired smooth function using the active control method and backstepping technique respectively.
Methods: Active control and recursive backstepping methods as well as Fourth-order Runge-Kutta algorithm was employed in all the simulations. In this work, the active control method has been applied to synchronize the chaotic Euler's equations for a rigid body evolving from different initial conditions. The control functions have been designed by means of Recursive Backstepping based on Lyapunov stability theory to control and track the chaotic system to a desired function.
Results: The results obtained show that the error state variables move chaotically with time initially when the controllers are deactivated and when the controllers are switched on at $t = 6$ s synchronization of the two systems evolving from different initial conditions is achieved. The state variable stabilize at the equilibrium point for $f(t) = 30\cos(0.05t)$. The results also showed that the designed controllers are effective in stabilization and tracking to any desired smooth function $f(t)$ of the chaotic system.
Conclusion: The control, tacking and synchronization of Euler's equations of rigid body was achieved using the backstepping technique and active control method respectively and this suggests the possibility for communication using chaotic wave forms as carriers, perhaps with application to secure communication.
Keywords: Euler's equation, Dynamical systems, Active Control, Chaos, Tracking, Synchronization.

All co-authors agreed to have their names listed as authors.

1. INTRODUCTION:

Edward Ott, Celso Grebogi and James Yorke (OGY) in [1] were the first to introduce chaos control, while the presentation of synchronization of chaotic systems was by Pecora and Carroll [2] in the same year. Thereafter, chaos control and synchronization has received increased attention. Several research have been carried out to develop chaos control and synchronization frameworks for dynamical systems due to its practical applications in science, engineering, biological sciences and to mention but a few.

In recent times, many systems have been developed to describe real life situations and the rigid body has evolved over time as one of the systems in this area. Due to its applicability, the regular and chaotic motions in applied dynamics of a rigid body has been studied by [3], where they stated that periodic and regular motions, having a predictable functioning mode, play an important role in many problems of dynamics. Therein, the structure of phase space was investigated as well as the phase trajectories of the motion which were constructed by a numerical implementation of the Poincare point map method. The transition to chaos in the phase portrait of a restricted problem of rotation of a rigid body with a fixed point has been studied by [4] and they went further to show that - two interrelated mechanisms responsible for chaotification are the growth of the homoclinic structure and the development of cascades of period doubling bifurcations.

In the works by [5, 6], the adaptive synchronization in chaotic rigid body motions were studied, where the sufficient stability criterion is derived for global synchronization of two chaotic rigid body motions with linear feedback control. In the work, the proposed scheme can be implemented without requiring the upper bound of the trajectory of the chaotic system. In the work of Laoye et al [7], chaos control and reduced-order synchronization of the rigid body was investigated.

In order to achieve chaos control and synchronization of chaotic systems, many control techniques have been developed and utilized such as active control [8,9,10,], backsteping technique [11,12], sliding mode control [13,14], etc.

In this work, we investigated the chaos control, tracking and synchronization of the Euler's rigid body. In the work by [7], tracking of the rigid body was not investigated and the synchronization is of reduced-order constituting the rigid body (3D) and a second-order Duffing oscillator whereas we considered the synchronization of the identical rigid body system of same order. The chaos control was of the same order and it was extended to tracking in our work.

The work is organized as follows: in the next section is the system description which is followed by designing the active control method controllers for synchronization of the rigid body of same order. Section 4 presents the chaos control of the rigid body which is followed by the tracking control in Section 5 and the paper is concluded in Section 6.

2. SYSTEM DESCRIPTION

The Euler's equations for the free rotation of rigid body was converted to chaotic system by using chaos anti-control and chaotic sequences were produced. [15]

Chaos control can suppress or eliminate chaotic dynamical behaviour. Chaotic anti-control through external input or adjustment or adjustment to internal parameters, mainly results in the original non-chaotic system becoming chaos or the chaos of the original system becoming stronger. In order to implement the control and anti-control of chaos the controller should be designed as simply as possible in order to ensure low-cost, easy realization and convenient use. The creation of a chaos generator to implement chaotic control is a problem for engineering design. A simple and strict chaos controller demands an equivalent level of competency in mathematics and the capability in the engineering design. [16, 17]

Feedback control is one of the basic methods for the control and anti-control of chaos. The linear feedback controller is the simplest controller that can be used to implement chaotic anti-control. Using Lyapunov exponents of control or anti-control of chaos is one of a number of effective ways to describe chaotic properties of non-linear systems. The number of Lyapunov exponents is the same as the dimension *n* of the state space of the system. If one of the Lyapunov exponents is greater than zero, the system is chaotic, and if at least two of the Lyapunov exponents are positive, the system is hyper-chaotic. The greater the number of positive Lyapunov exponents, the higher the degree of unpredictability in the system.

The Euler's equations of free rotation of rigid body are given by [18, 19] as:

$$\begin{cases} I_1\dot{w}_1 = (I_2 - I_3)w_2w_3 + M_1 \\ I_2\dot{w}_2 = (I_3 - I_1)w_1w_3 + M_2 \\ I_3\dot{w}_3 = (I_1 - I_2)w_1w_2 + M_3 \end{cases} \qquad (1)$$

Where $I_1, I_2, and\ I_3$ are main moments of inertia and $w_1, w_2, and\ w_3$ are angular velocities of the spindle and $M_1, M_2, and\ M_3$ are the imposed torques, respectively. Through the linear feedback system, a non-chaotic free rotation system of rigid body is transformed into a chaotic system. Because the angular velocities are changing parameters of the system (note: The mechanism synthesis only requires that chaotic sequences be produced without considering the parameter direction. It is easy to detect the angular velocities in the engineering system), the imposed moments are feedback linearly by the angular velocities, that is, $M = Aw$, where

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

Suppose, $w_1 = x,\ w_2 = y,\ w_3 = z,\ a = a_{11}/I_1$, $b = a_{22}/I_2$, and $c = a_{33}/I_3$

Equation (1) is transformed into equation (2) as:

$$\begin{cases} \dot{x} = \frac{I_2 - I_3}{I_1} yz + ax \\ \dot{y} = \frac{I_3 - I_1}{I_2} xz + by \\ \dot{z} = \frac{I_1 - I_2}{I_3} xy + cz \end{cases} \qquad (2)$$

The conditions whereby the system produces chaos are as follows:

$$\text{(i)} \begin{cases} a > 0, b < 0, c < 0, and\ 0 < a < -(b+c)\ or \\ b > 0, a < 0, c < 0, and\ 0 < b < -(a+c)\ or \\ c > 0, a < 0, b < 0, and\ 0 < c < -(a+b) \end{cases}$$

$$\text{(ii)} \begin{cases} \frac{I_2 - I_3}{I_1} < 0,\ \frac{I_3 - I_1}{I_2} > 0,\ \frac{I_1 - I_2}{I_3} > 0,\ that\ is, I_3 > I_1 > I_2\ or \\ \frac{I_2 - I_3}{I_1} > 0,\ \frac{I_3 - I_1}{I_2} < 0,\ \frac{I_1 - I_2}{I_3} < 0,\ that\ is, I_2 > I_1 > I_3 \end{cases}$$

We now select $I_3 = 3I_o, I_1 = 2I_o, I_2 = I_o$ (to meet $I_3 > I_1 > I_2$), also a = 5, b = -10, c = -3.8 (to meet $a > 0, b < 0, c < 0$ and $0 < a < -(b+c)$,)

Equation (2) is transformed into equation (3) as:

$$\begin{cases} \dot{x} = -yz + ax \\ \dot{y} = xz + by \\ \dot{z} = \frac{1}{3} xy + cz \end{cases} \qquad (3)$$

In equation (3), x, y, z are the states and (*a, b, c*) are system parameters. We show that it is chaotic for the parameters:

$$a = 5, b = -10, c = -3.8. \qquad (4)$$

Using Wolf's algorithm [20], the Lyapunov exponents of system (3) are obtained for the parameter values as in (4) and $X(0) = (0.2, 0.2, 0.2)$ for $T = 2E3$ seconds as

$$L_1 = 0.6351, L_2 = 0, L_3 = -9.4317 \qquad (5)$$

Clearly, system (3) is chaotic since it has a positive Lyapunov exponent L_1, see figures (1) and (2). By adding all the Lyapunov exponents in (5), we get their sum as: $L_1 + L_2 + L_3 = -8.7966 < 0$, showing that system (3) is dissipative and it has a strange chaotic attractor. The Kaplan-Yorke fractal dimension of system (3) is calculated as:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0673$$

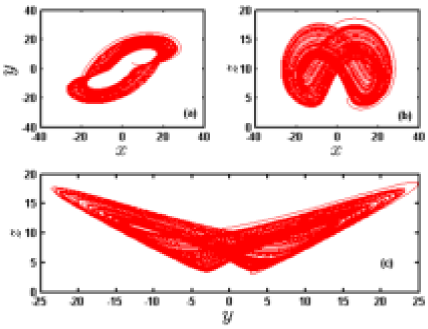


Figure 1 Phase Portraits: (a) xy-plane, (b) xz-plane, (c) yz-plane

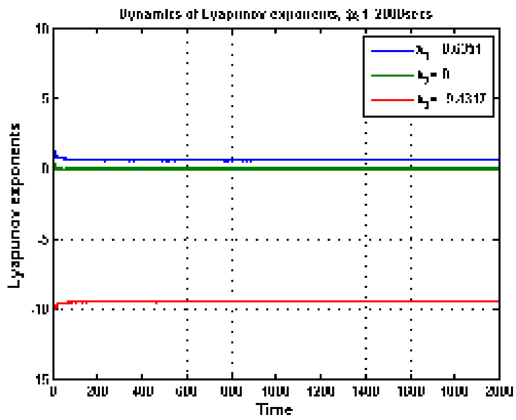


Figure 2: Lyapunov Exponents for system equation 3

2.1. Designed of active control for synchronization

We consider and rewrite system (3) as the drive system as:

$$\begin{cases} \dot{x}_1 = -x_2 x_3 + a x_1 \\ \dot{x}_2 = x_1 x_3 + b x_2 \\ \dot{x}_3 = \frac{1}{3} x_1 x_2 + c x_3 \end{cases} \qquad (6)$$

And the response system as:

$$\begin{cases} \dot{y}_1 = -y_2 y_3 + a y_1 + u_1 \\ \dot{y}_2 = y_1 y_3 + b y_2 + u_2 \\ \dot{y}_3 = \frac{1}{3} y_1 y_2 + c y_3 + u_3 \end{cases} \qquad (7)$$

Where $u_i (i = 1,2,3)$ are active control functions to be determined; defining the error states for the states variables as:

$$e_1 = y_1 - x_1, e_2 = y_2 - x_2, e_3 = y_3 - x_3 \qquad (8)$$

And following the procedure of active control design, we subtract system (6) from system (7) and using the definition in equation (8), we obtain the error dynamics as equation given by:

$$\begin{cases} \dot{e}_1 = -e_2(e_3 + x_3) - x_2 e_3 + a e_1 + u_1 \\ \dot{e}_2 = e_1(e_3 + x_3) + x_1 e_3 + b e_2 + u_2 \\ \dot{e}_3 = \frac{1}{3}[e_1(e_2 + x_2) + x_1 e_2] + c e_3 + u_3 \end{cases} \qquad (9)$$

Redefining the control functions as follows:

$$\begin{cases} u_1 = v_1 + [e_2(e_3 + x_3) + x_2 e_3] \\ u_2 = v_2 - [e_1(e_3 + x_3) + x_1 e_3] \\ u_3 = v_3 - \frac{1}{3}[e_1(e_2 + x_2) + x_1 e_2] \end{cases} \quad (10)$$

The error dynamics equation (9) becomes:

$$\begin{cases} \dot{e}_1 = ae_1 + v_1 \\ \dot{e}_2 = be_2 + v_2 \\ \dot{e}_3 = ce_3 + v_3 \end{cases} \quad (11)$$

In the active control method, we choose a constant matrix A which will control the error dynamics (11) such that:

$$[v_1, v_2, v_3]^T = A[e_1, e_2, e_3]^T \quad (12)$$

Several choice of A can satisfy system (12). Here we choose the following matrix that satisfy the Routh-Hurwtiz criteria for stability of the synchronized state:

$$A = \begin{pmatrix} \lambda_1 - a & 0 & 0 \\ 0 & \lambda_2 - b & 0 \\ 0 & 0 & \lambda_3 - c \end{pmatrix} \quad (13)$$

Which immediately yields the control functions:

$$\begin{cases} u_1 = (\lambda_1 - a)e_1 + [e_2(e_3 + x_3) + x_2 e_3] \\ u_2 = (\lambda_2 - b)e_2 - [e_1(e_3 + x_3) + x_1 e_3] \\ u_3 = (\lambda_3 - c)e_3 - \frac{1}{3}[e_1(e_2 + x_2) + x_1 e_2] \end{cases} \quad (14)$$

Provided the eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ are negative definite. Here, we have chosen $\lambda_1, \lambda_2, \lambda_3) = (-1, -1, -1)$ for simplicity.

3. NUMERICAL SIMULATIONS

By using fourth-order Runge-Kutta algorithm with initial conditions $(x_1, x_2, x_3) = (-5,5,5)$, $(y_1, y_2, y_3) = (10,10,-11)$, a time step of 0.001 and fixing the parameter as in figure (1) to ensure chaotic dynamics of the state variable, we solved systems; (6), (7) and (9) with the controllers $u_i(i = 1,2,3)$ as defined in (14). The results obtained show that the error state variables move chaotically with time when the controllers are deactivated and when the controllers are switched on at t = 6 seconds as shown in figure (3a) to (3c), while the error state variables converges to the origin and thereby guaranteeing the synchronization of the system (6) and (7). See Figure (3d).

Figure (3a) to (3c); Complete synchronization of system (6) and (7) with and

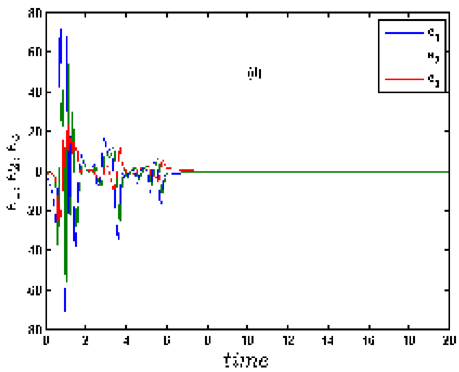


Figure (3d) Time-history of synchronization error between systems (6) and (7) with $(x_1, x_2, x_3) = (-5,5,5)$ and $(y_1, y_2, y_3) = (-10,10,11)$

4 CHAOS CONTROL BASED ON RECURSIVE BACKSTEPPING.

Consider an autonomous system in ''strict-feedback'' form as follows:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) \\ \dot{x}_2 &= f_2(x_1, x_2, x_3) \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, \dots, x_n) + f_{n+1}(t) \end{aligned} \quad (15)$$

Where $[x_1, x_2, \dots x_n]^T \in \mathbb{R}^n$ are the state variables of the system, f_1 is a linear function, $f_i(i = 1,2, \dots, n+1)$ are nonlinear functions and $f_{n+1}(t)$ is a periodic function of time. To control the chaotic system in the form of (15) as follows:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + u_1 \\ \dot{x}_2 &= f_1(x_1, x_2, x_3) + u_2 \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, \dots, x_n) + f_{n+1}(t) + u_n \end{aligned} \quad (16)$$

Where $u_i(t)(i = 1,2, \dots, n)$ are control functions. Now consider a known, bounded and smooth reference model given as: $\dot{x}_{ri} = f_{ri}(x_r, t), 1 \le i \le m, n \le m$ where $[x_{r1}, x_{r2}, \dots x_{rm}]^T \in \mathbb{R}^m$ are the state variables: $f_{ri}(.), (i = 1,2, \dots, m)$ are known nonlinear functions with their j^{th} derivatives uniformly bounded in t. the objective is to design recursive backstepping controllers for system (15) that guarantees global stability and forces the output $x(t)$ of system (15) to asymptotically track the output $x_r(t)$ of reference model that is: $|x - x_r| \to 0, as\ t \to \infty$. The new chaotic system to be controlled is given by

$$\begin{cases} \dot{x}_1 = -x_2 x_3 + ax_1 + u_1(t) \\ \dot{x}_2 = x_1 x_3 + bx_2 + u_2(t) \\ \dot{x}_3 = \frac{1}{3}x_1 x_2 + cx_3 + u_3(t) \end{cases} \quad (17)$$

Where $u_i(t)(i = 1,2,3)$control functions to be determined, to obtain the controllers, $u_i(t)$, we defined the error states as:

$e_1 = x_1 - x_{1d}, e_2 = x_2 - x_{2d}, e_3 = x_3 - x_{3d}$

where x_{1d}, x_{2d}, x_{3d} are the reference outputs which are recursively defined as follows: $x_{1d} = 0$

$$x_{2d} = c_1 e_1 \quad (18)$$
$$x_{3d} = c_2 e_1 + c_3 e_2$$

Where $c_i(i = 1,2,3)$ are arbitrary control parameters to be chosen later. With equation (17) and: $e_1 = x_1 - x_{1d}, e_2 = x_2 - x_{2d}, e_3 = x_3 - x_{3d}$, we obtain the error dynamics given by:

$$\dot{e}_1 = ae_1 - (e_2 + c_1e_1)(e_3 + c_2e_1 + c_3e_2) + u_1(t)$$
$$\dot{e}_2 = (e_3 + c_2e_1 + c_3e_2) + b(e_2 + c_1e_1) - c_1\dot{e}_1 + u_2(t) \quad (19)$$
$$\dot{e}_3 = \frac{1}{3}e_1(e_2 + c_1e_1) + c(e_3 + c_2e_1 + c_3e_2) - (c_2\dot{e}_1 + c_3\dot{e}_2) + u_3(t)$$

Consider the Lyapunov function as:

$$V = \frac{1}{2}\sum_{i=1}^{3} e_i^2 \quad (20)$$

The time derivative of (20) is:

$$\dot{V} = \sum_{i=1}^{3} e_i\dot{e}_i = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \quad (21)$$

$$\dot{V} = e_1[ae_1 - (e_2 + c_1e_1)(e_3 + c_2e_1 + c_3e_2) + u_1(t)]$$
$$+ e_2[(e_3 + c_2e_1 + c_3e_2) + b(e_2 + c_1e_1) - c_1\dot{e}_1 + u_2(t)] \quad (22)$$
$$+ e_3[\frac{1}{3}e_1(e_2 + c_1e_1) + c(e_3 + c_2e_1 + c_3e_2) - (c_2\dot{e}_1 + c_3\dot{e}_2) + u_3(t)]$$

$$u_1 = -(ae_1 - e_2e_3) - e_1$$
$$u_2 = -(be_2 + e_1e_3) - e_2 \quad (23)$$
$$u_3 = -[ce_3 + \frac{1}{3}e_1e_2] - e_3$$

And $c_i(i = 1,2,3) = 0$, then

$$\dot{V} = -(e_1^2 + e_2^2 + e_3^2) < 0 \quad (24)$$

Is negative definite and according to LaSalle-Yoshizawa theorem the equilibrium (0,0,0) of system (20) is asymptotically stable. The controller $u(t)$ does not change the equilibrium of system (20). That is (0, 0, 0) is still in equilibrium. Consequently the new chaotic system is stabilised at the origin under the controller (23).

4.1 RESULTS COMPUTATIONS

In figure (3a) to (3c), we present numerical results to illustrate the effectiveness of the controllers. The controllers have been activated at $t \geq 20$. It is obvious that the system has been stabilized to the desired equilibrium point.

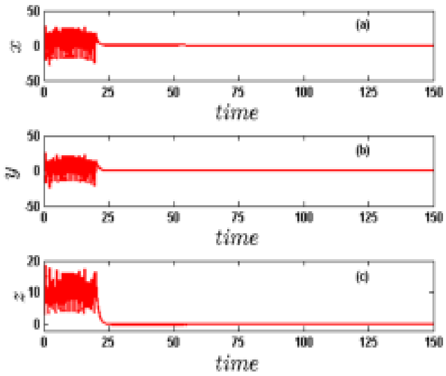


Figure 3 Time response of the state

4.2 TRACKING CONTROL OF THE CHAOTIC SYSTEM.

We present recursive backstepping control for tracking of the chaotic system, to track to a smooth function of time $f(t)$.

$$\begin{cases} \dot{x}_1 = -x_2x_3 + ax_1 + u_1(t) \\ \dot{x}_2 = x_1x_3 + bx_2 + u_2(t) \\ \dot{x}_3 = \frac{1}{3}x_1x_2 + cx_3 + u_3(t) \end{cases} \quad (25)$$

Where $u_i(t)(i = 1,2,3)$ control functions to be designed, to obtain the controllers, $u_i(t)$, we defined the error states as:

$e_1 = x_1 - x_{1d}, e_2 = x_2 - x_{2d}, e_3 = x_3 - x_{3d}$

where x_{1d}, x_{2d}, x_{3d} are the reference outputs which are recursively defined as follows: $x_{1d} = f(t)$

$$x_{2d} = c_1e_1 \quad (26)$$
$$x_{3d} = c_2e_1 + c_3e_2$$

Where $c_i(i = 1,2,3)$ are arbitrary control parameters to be chosen approximately. With equation (25) and: $e_1 = x_1 - x_{1d}, e_2 = x_2 - x_{2d}, e_3 = x_3 - x_{3d}$, we obtain the error dynamics given by:

$$\dot{e}_1 = -(e_2 + c_1e_1)(e_3 + c_2e_1 + c_3e_2) + a(e_1 + f(t)) - \dot{f(t)} + u_1(t)$$
$$\dot{e}_2 = (e_1 + f(t))(e_3 + c_2e_1 + c_3e_2) + b(e_2 + c_1e_1) - c_1\dot{e}_1 + u_2(t) \quad (27)$$
$$\dot{e}_3 = \frac{1}{3}(e_1 + f(t))(e_2 + c_1e_1) + c(e_3 + c_2e_1 + c_3e_2) - (c_2\dot{e}_1 + c_3\dot{e}_2) + u_3(t)$$

Consider the Lyapunov function as:

$$V = \frac{1}{2}\sum_{i=1}^{3} k_ie_i^2 \quad (28)$$

The time derivative of (28) is:

$$\dot{V} = \sum_{i=1}^{3} e_i\dot{e}_i = k_1e_1\dot{e}_1 + k_2e_2\dot{e}_2 + k_3e_3\dot{e}_3 \quad (29)$$

To satisfy the condition for asymptotic stability of the error system (27) necessary for tracking, such that $\dot{V} = -\frac{1}{2}\sum_{i=1}^{3} k_ie_i^2 < 0$ we substitute (27) into (29) with the choice of controllers as follows:

$\dot{e}_1 = -e_1, \dot{e}_2 = -e_2, \dot{e}_3 = -e_3$ so that, the controllers can be written as:

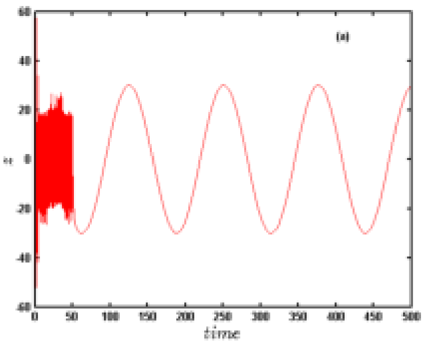
$$u_1 = (e_2 + c_1 e_1)(e_3 + c_2 e_1 + c_3 e_2) - a\big(e_1 + f(t)\big) + \dot{f(t)} - e_1$$
$$u_2 = -\big(e_1 + f(t)\big)(e_3 + c_2 e_1 + c_3 e_2) - b(e_2 + c_1 e_1) + c_1 \dot{e}_1 - e_2 \qquad (30)$$
$$u_3 = -\tfrac{1}{3}\big(e_1 + f(t)\big)(e_2 + c_1 e_1) - c(e_3 + c_2 e_1 + c_3 e_2) + (c_2 \dot{e}_1 + c_3 \dot{e}_2) - e_3$$

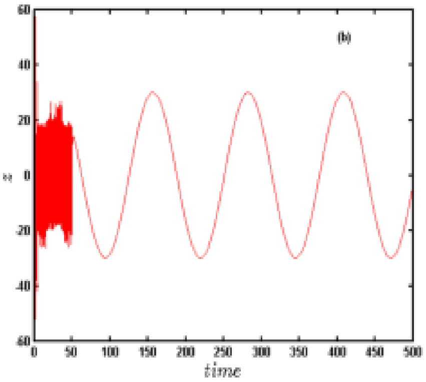
We choose $e_1 = e_2 = e_3 = 1$, from our observation that the numerical simulation of system (25) be effectively controlled. Equation (30) reduces to:

$$u_1 = (e_2 + e_1)(e_3 + e_1 + e_2) - a\big(e_1 + f(t)\big) + \dot{f(t)} - e_1$$
$$u_2 = -\big(e_1 + f(t)\big)(e_3 + e_1 + e_2) - b(e_2 + e_1) + \dot{e}_1 - e_2 \qquad (31)$$
$$u_3 = -\tfrac{1}{3}\big(e_1 + f(t)\big)(e_2 + e_1) - c(e_3 + e_1 + e_2) + (\dot{e}_1 + \dot{e}_2) - e_3$$

4.3 NUMERICAL SIMULATIONS.

We verify the effectiveness of the proposed scheme, the fourth-order Runge-Kutta algorithm is applied with the initial conditions $(x_1, x_2, x_3) = (10,10,10)$, and a time step of 1e-3 and fixing the parameter values as in Figure (1) to ensure chaotic dynamics of the state variables, we solve system (25) with controllers $u_i(t)(i = 1,2,3)$ as defined as in (31). The result obtained show that the state variables move chaotically with time when the controllers are deactivated and when the controllers are switched on at $t = 50\ s$ the state variable stabilize at the equilibrium point for $f(t) = 30\cos(0.05t)$ as shown in figure (4). The results showed that the designed controllers (31) are effective in stabilization and to track any desired smooth function $f(t)$ of chaotic system.





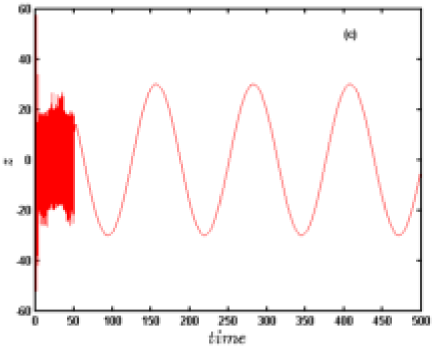


Figure 4 (a) to (c): Time responses of the state variables $(\mathbf{x_1, x_2, x_3})$ for chaotic system with the control $\mathbf{u_i(t)}$ activated at $\mathbf{t = 50\ s}$ to track $(\mathbf{t}) = \mathbf{30\cos(0.05t)}$

5. Conclusion:

In this work, the active control method has been applied to control and synchronize chaotic Euler's equations for a rigid body. Also control functions have been designed by means of Recursive Backstepping based on Lyapunov stability theory to control, track and synchronize the two identical chaotic systems evolving from different initial conditions. Numerical simulations are given to demonstrate the effectiveness of the proposed controllers. Control and synchronization of Euler's equations of rigid body suggests the possibility for communication using chaotic wave forms as carriers, perhaps with application to secure communication.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

AUTHORS' CONTRIBUTIONS

Cornelius Ogabi prepared the paper, carried out all the simulations, wrote the protocol and the first draft of the manuscript. Babatunde Idowu wrote the introduction and other authors managed the literature searches, read and approved the final manuscript.

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