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ORIGINAL RESEARCH

Projective Synchronization of a 3-D Chaotic System with Quadratic and Quartic Nonlinearities.

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Abstract:
Introduction: Chaos is a state of dynamical systems whose apparently random states of disorder and irregularities are often governed by deterministic laws that are highly sensitive to initial conditions. In this work, the projective synchronization of two identical three-dimensional chaotic system with quadratic and quartic nonlinearities was considered as well as the equilibrium and stability analysis of the system. The projective synchronization with same and different scaling factors was carried out in order to establish its synchronization.
Aim: :To achieve projective synchronization of two identical three-dimensional chaotic system with quadratic and quartic nonlinearities synchronizing to a scaling factor and also present the equilibrium and stability analysis of the system.
Methods: We employed the adaptive synchronization technique to achieve projective synchronization of the system (master and slave) with different scaling factors,β and the fourth-order Runge-Kutta algorithm was used for numerical solutions.
Results: In this work, the projective synchronization of two identical three-dimensional systems with quadratic and quartic nonlinearities was achieved with the same and different scaling factors,β. The equilibrium and stability analysis of the system was also presented.
Conclusion: The investigated projective synchronization behaviour of two identical three-dimensional system with two nonlinearities (quadratic and quartic) was achieved for cases where the scaling factor is the same and when different. This shows that projective synchronization can be achieved for systems with varying nonlinearities even when the scaling factor is different and this suggests its use in communication using chaotic wave forms as carriers, perhaps with a view to securing communication.

Keywords: Projective Synchronization, nonlinearities, scaling factors, Runge-Kutta, Quadratic and quartic nonlinearities.

All co-authors agreed to have their names listed as authors.

1 INTRODUCTION

Chaos synchronization is an extension of the concept of chaos control. Nowadays, the control and synchronization of the chaotic systems have received a lot of attention by researchers because of its unpredictable complex behaviour. The ultimate objective of chaos synchronization is to design a feedback controller for the follower chaotic system such that the drive system tracks the trajectories of the driven chaotic system as time goes to infinity. However, the challenges occur when the chaotic systems are exposed to some uncertainty, unknown system parameters and have different initial values. Then, some actions have to be taken in order to stabilize and to improve synchronization. In 1963, Lorenz discovered the famous Lorenz chaotic system [1]. After that, chaotic systems have been researched extensively, such as the Lu system [2, 3, 4] the Chen system [5] and the Rössler system [6] new chaotic finance system [7] and so on.

Chaotic systems must have one positive Lyapunov exponents while Hyperchaotic systems must have at least two positive Lyapunov exponents, and the dimension must be four or more [8]. Chaotic systems are suitable for some engineering applications, such as chemical reactions, electric circuits [9], cryptography [10, 11] and fluid dynamics and secure communication [12, 13, 14].

Chaos synchronization is another fascinating concept. Pecora and Carroll proposed a drive-response chaotic synchronization scheme in 1990 [15, 16] and realized the synchronization of two chaotic systems in the circuit, which promoted the theoretical study of chaotic synchronization and chaos control. Since then, many effective chaotic synchronization methods have emerged, such as complete synchronization [17, 18] generalized synchronization [19], phase synchronization [20], lag synchronization [21, 22, 23], projective synchronization [24, 25] anticipating synchronization [26] and exponential synchronization [27, 28]. In recent years, the synchronization of chaotic fractional differential systems [29, 30, 31] has attracted more and more attention because of its potential applications in secure communication and control processes [32, 33, 34, 35].
The research on projective synchronization [36] has received extensive attention from researchers in recent years. Projective synchronization suffice that under certain conditions, the output of the coupled drive system and the response system state is not only phase locked, but the amplitude of each corresponding state also evolve according to a certain scale factor relationship. The method has been observed and discussed in coupled integer order chaotic systems [37, 38, 39, 40].

Modified projective synchronization, was considered by Li [41, 42] in which the response of synchronized dynamical states can synchronize up to a constant matrix. The modified projective synchronization was an extension of generalized projective synchronization.

There is also fuzzy adaptive controller for achieving an appropriate generalized projective synchronization (GPS) of two incommensurate fractional-order chaotic systems. The master system and the slave system, are assumed to be with non-identical structure, external dynamical disturbances, uncertain models and distinct fractional-orders. The adaptive fuzzy systems are used for estimating some unknown nonlinear functions, see [43]. In achieving synchronization many methods have been proposed such as active control [44, 45, 46, 47, 48], backstepping [49, 50], adaptive backstepping [51, 52, 53, 54], sliding mode, and so on.

The other parts of the article is organized as follows: In section 2, a new three-dimensional system with two nonlinearities is constructed and the dynamical behaviours of the system are discussed, such as attractor, dissipativity and equilibrium points. In section 3, the projective synchronization scheme of the new three-dimensional system with two nonlinearities is designed and some numerical simulations are completed. In section 4, some conclusions.

2 SYSTEM DESCRIPTION, EQUILIBRIUM AND STABILITY

2.1 Systems Description

Vaidyanathan et al. [55, 56, 57] propose a new three-dimensional system with two nonlinearities given by the following nonlinear differential equation, (1):-

$$\begin{aligned} \dot{x_1} &= a(x_2 - x_1) \\ \dot{x_2} &= x_1 x_3 \\ \dot{x_3} &= 50 - b x_1^4 - c x_3 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the states of the system and a, b, c are positive parameters, and control the dynamics of the system. With system (1) describes a self-excited chaotic attractor phase space plot as shown in figure 1. The initial condition is taken as $(2.6, 3.7, 1.4)$. Also, the slave system is described by the controlled chaotic system

$$\begin{aligned} \dot{y_1} &= a(y_2 - y_1) + u_1 \\ \dot{y_2} &= y_1 y_3 + u_2 \\ \dot{y_3} &= 50 - b y_1^4 - c y_3 + u_3 \end{aligned} \tag{2}$$

Where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are active controllers to be designed.

The divergence of system (1) is in the form

$$\nabla V = \frac{d\dot{x_1}}{dx_1} + \frac{d\dot{x_2}}{dx_2} + \frac{d\dot{x_3}}{dx_3} \tag{3}$$

which is $-(a + 0 + c) = (3 + 0 + 1) = -4 < 0$

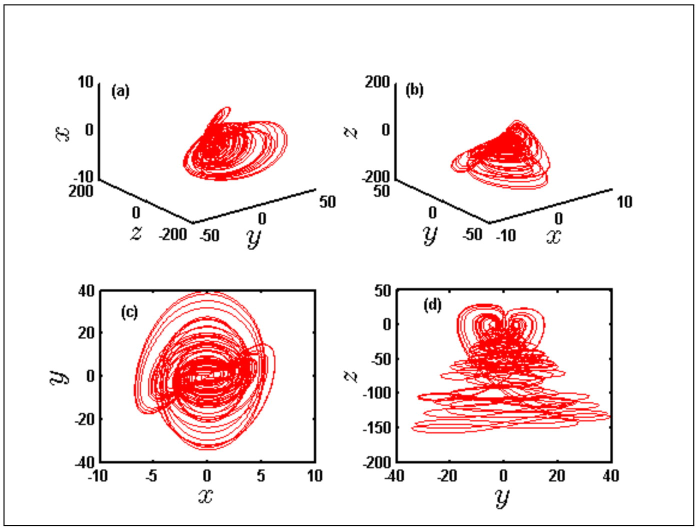

Figure 1: Phase Portrait of system (1): (a), (x_2, x_3, x_1) plane; (b), ($x_1, x_2, x_3)$ plane; (c), (x_1, x_2) plane; (d), (x_2, x_3) plane.

2.2 Equilibrium and Stability Analysis

The equilibrium of the system (1) can be obtained by solving the following equations

$$\begin{aligned} a(x_2 - x_1) &= 0 \\ x_1 x_3 &= 0 \\ 50 - bx_1^4 - cx_3 &= 0 \end{aligned} \tag{4}$$

It is easy to find that system (4) has only one trivial equilibrium point . The Jacobian matrix is given by

$$J = \begin{bmatrix} -a & a & 0 \\ 50 & 0 & 0 \\ 0 & 0 & -c \end{bmatrix} \tag{5}$$

Taking $x_1 = 0$, then $x_2 = 0$ and $x_3 = 50$ The eigenvalues of the Jacobian matrix J are: $\lambda_1 = -13.8390$, $\lambda_2 = 10.8390$, $\lambda_3 = -1.0000$ The eigenvalues are real and have opposite signs, this shows that the equilibrium of the system is a saddle point.

3 METHODS:

3.1 Projective synchronization with the same scaling factors

Our objective is to find suitable controllers$u_i, (i = 1, 2, 3)$ to ensure the drive system (1) and the response system (2) approach projective synchronization with a scaling factor, β .The projective synchronization error vector is defined by $e_i = x_i - \beta y_i, (i = 1, 2, 3)$.

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e_1} &= a(e_2 - e_1) - \beta u_1 \\ \dot{e_2} &= x_1 x_3 - \beta y_1 y_3 - \beta u_2 \\ \dot{e_3} &= 50(1 - \beta) - b(x_1^4 - \beta y_1^4) - ce_3 - \beta u_3 \end{aligned} \tag{6}$$

With different initial conditions, trajectories of the systems (1) and (2) without controllers will diverge exponentially due to the butterfly effect. Nevertheless, when suitable controllers are designed, the two chaotic systems will approach projective synchronization for any initial conditions.
In order to achieve this objective, we choose the following controllers

$$\begin{aligned} u_1 &= \frac{1}{\beta}\left[a(e_2 - e_1) + e_1\right] \\ u_2 &= \frac{1}{\beta}\left[x_1 x_3 - \beta y_1 y_3 + e_2\right] \\ u_3 &= \frac{1}{\beta}\left[50(1 - \beta) - b(x_1^4 - \beta y_1^4) - ce_3 + e_3\right] \end{aligned} \tag{7}$$

Systems (1) and (2) will achieve projective synchronization asymptotically with the controllers (7) with a scaling factor.
Proof. Choose the following Lyapunov function

$$V(e) = e^T P e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{8}$$

Where, $P = dia\,[1, 1, 1]$ The derivative of the Lyapunov function $V(e)$ with respect to time is:

$$\begin{aligned} \dot{V}(e) &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} \\ &= e_1(ae_2 - ae_1 - \beta u_1) + e_2(x_1 x_3 - \beta y_1 y_3 - \beta u_2) \\ &+\ e_3(50(1 - \beta) - b(x_1^4 - \beta y_1^4) - ce_3 - \beta u_3) \end{aligned} \tag{9}$$

Substitute equation (7) into equation (9)

$$\begin{aligned} \dot{V}(e) &= e_1(-e_1) + e_2(-e_2) + e_3(-e_3) \\ \dot{V}(e) &= -(e_1^2 + e_2^2 + e_3^2) \\ \dot{V}(e) &< 0 \end{aligned}$$

Therefore,$\dot{V}(e)$ is negative definite. And according to the Lyapunov stability theory, the error dynamical system (6) is asymptotically stable at the origin. Therefore, projective synchronization between the system (1) and (2) is achieved with the controllers (7). The proof is now complete.

3.1.1 Result discussion and output of results

Next, we perform numerical simulations to show the feasibility and effectiveness of the designed controller. Choose the scaling factor, $\beta = 5$. The 4th order Runge–Kutta method is employed to integrate the differential equations. The initial conditions of the drive system and the response system are $(2.6, 3.7, 1.4)$ and $(1.4, 5.9, 2.1)$, respectively. The synchronization of systems (1) and (2) is achieved using the controllers (7). The result shows the state variable moves chaotically with time when the controller is de-activated. This shows that these systems depend sensitively on initial condition in the absence of controller, which is a major character of a chaotic system. We depict in Figure 2 that the variable errors do not synchronize with time in the absence of controllers but when the controllers are introduced at $t \geqslant 5.5$ the variable errors synchronize with time. This is confirmed by the synchronization quality e, given by $e = \sqrt{(e_1^2 + e_2^2 + e_3^2)}$ as shown in Figure 3.

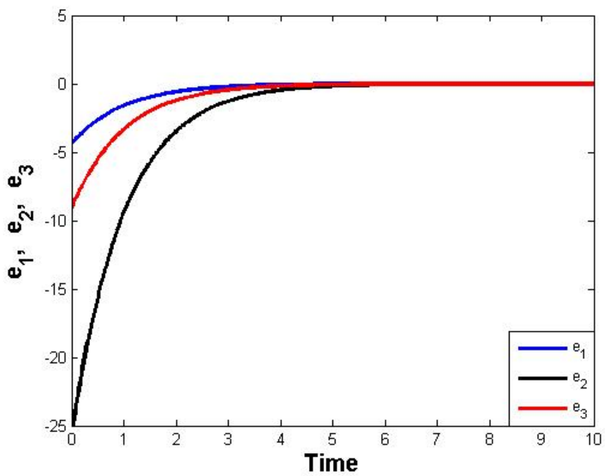

Figure 2: Error dynamics between systems (1) and (2) with the controllers deactivated for $0 < t = 5.5$ and activated for $t \geqslant 5.5$.

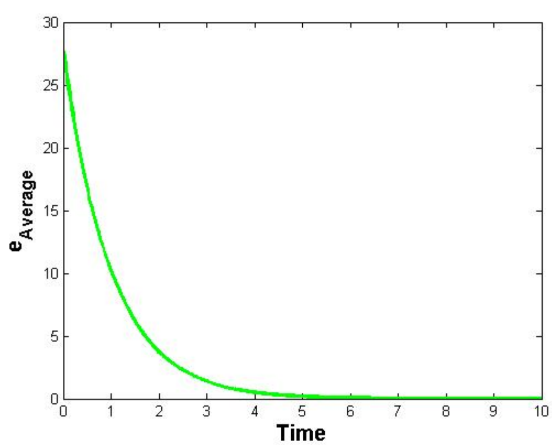

Figure 3: Synchronization quality between systems(1) and (2).

3.2 Projective synchronization with the different scaling factors

In this section, we present projective synchronization with different scaling factors which implies that the three state variables of the drive system are in proportion to that of the response system with three different scaling factors $\beta_1, \beta_2, \beta_3$ respectively. In other word, there exists a constant matrix $\beta = dia(\beta_1, \beta_2, \beta_3)$ such that $\lim_{t\to\infty} |x - \beta y| = 0$ This synchronization form called 'modified projective synchronization' in [58, 59, 60] which has been considered in chaotic systems . We choose exactly the same master–slave system described in the last section, and define the error vectors as $e_i = x_i - \beta_i y_i, (i = 1, 2, 3)$. Hence, the error dynamical system is obtained as

$$\begin{aligned} \dot{e_1} &= a(x_2 - \beta_1 y_2) - ae_1 - \beta_1 u_1 \\ \dot{e_2} &= x_1 x_3 - \beta_2 y_1 y_3 - \beta_2 u_2 \\ \dot{e_3} &= 50(1 - \beta_3) - b(x_1^4 - \beta_3 y_1^4) - ce_3 - \beta_3 u_3 \end{aligned} \tag{10}$$

Our focus is to design suitable controllers to achieve projective synchronization with different scaling factors between the drive systems (1) and the response system (2). Choosing the control functions $u_i, (i = 1, 2, 3)$ as

$$\begin{aligned} u_1 &= \frac{1}{\beta_1}\left[a(x_2 - \beta_1 y_2) - ae_1 + e_1\right] \\ u_2 &= \frac{1}{\beta_2}\left[x_1 x_3 - \beta_2 y_1 y_3 + e_2\right] \\ u_3 &= \frac{1}{\beta_3}\left[50(1 - \beta_3) - b(x_1^4 - \beta_3 y_1^4) - ce_3 + e_3\right] \end{aligned} \tag{11}$$

Proof: Systems (1) and (2) will achieve projective synchronization asymptotically with the controllers (11) with different scaling factors. *Proof*: Choose the following Lyapunov function

$$V(e) = e^T P e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{12}$$

Where $P = dia[1, 1, 1]$The derivative of the Lyapunov function $V(e)$ with respect to time is

$$\begin{aligned} \dot{V}(e) &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} \\ &= e_1(ax_2 - a\beta_1 y_2 - ae_1 - \beta_1 u_1) + e_2 x_1 x_3 - e_2(\beta_2 y_1 y_3 \\ &\quad + \beta_2 u_2) + e_3(50(1 - \beta_3) - b(x_1^4 - \beta_3 y_1^4) - ce_3 - \beta_3 u_3) \end{aligned} \tag{13}$$

Substitute equation (11) into equation (13)

$$\begin{aligned} \dot{V}(e) &= e_1(-e_1) + e_2(-e_2) + e_3(-e_3) \\ \dot{V}(e) &= -(e_1^2 + e_2^2 + e_3^2) \\ \dot{V}(e) &< 0 \end{aligned}$$

Therefore, the error dynamical system (10) is asymptotically stable at the origin according to the Lyapunov stability theory. And the drive system (1) and the response system (2) can approach projective synchronization with different scaling factors asymptotically with the controllers (11). The proof is now complete.

3.2.1 Result discussion and output of results

In order to verify the effectiveness of the proposed controllers in (11), numerical simulations are performed. Choose the scaling matrix $\beta = dia[1, 3, 5]$ the 4th order Runge–Kutta method is employed to integrate the differential equations. The initial conditions of the drive system and the response system are $(2.6, 3.7, 1.4)$ and $(1.4, 5.9, 2.1)$, respectively. Figure 4 shows that the variable errors do not synchronize with time in the absence of controllers but when the controllers are activated at $t \geq 5.5$ although the initial conditions are different. The state variables of the system (1) are in proportion to that of the system (2) with different scaling factors $1, 3, 5$, respectively. This is confirmed by the synchronization quality $e = \sqrt{(e_1^2 + e_2^2 + e_3^2)}$, given by as shown in Figure 5.

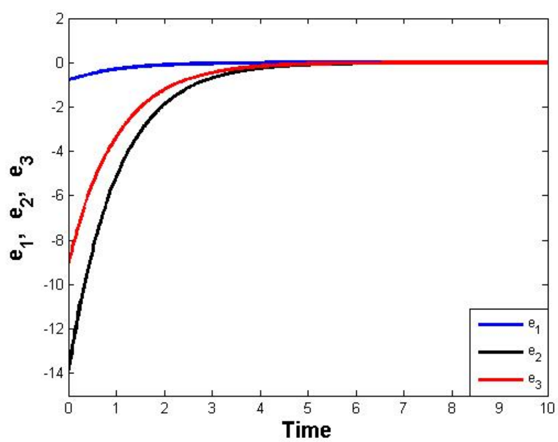

Figure 4: Error dynamics between systems (1) and (2) with the controllers deactivated for $0 < t = 5.5$ and activated for $t \geqslant 5.5$.

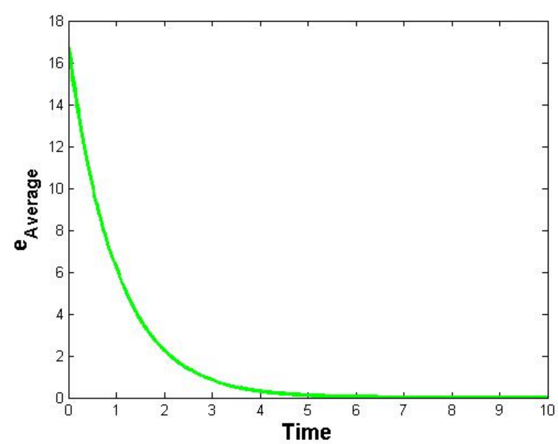

Figure 5: Synchronization quality between systems(1) and (2).

4 CONCLUSION

In this research work, we have observed projective synchronization behaviour of two identical new three-dimensional system with two nonlinearities equation (1). The conservative chaotic systems have the important property that they are volume conserving. Generaly, The Kaplan-Yorke dimension is calculated from $D_{KY} = 2+\frac{L_1+L_2}{|L_3|}$ for $equation(1)$ and it is found to be 2.8554 The high value of the Kaplan-Yorke dimension indicates the complexity of the chaotic system. The Lyapunov exponents of the 3-D novel chaotic system have been obtained as $L_1 = -1.0000, L_2 = -10.8352$ and $L_3 = 13.8356$. Also, the maximal Lyapunov exponent of the 3-D novel conservative chaotic system is $L_3 = 13.8356$. The phase portraits, figure (1) of the novel chaotic system were simulated using MATLAB. Generalized projective synchronization is a general type of synchronization which generalizes common types of synchronization such as complete synchronization (CS), anti-synchronization (AS), hybrid synchronization (HS), projective synchronization (PS), etc. The PS synchronization result was established using Lyapunov stability theory. Finally, we have investigated projective synchronization behaviour of two identical new three-dimensional system with two nonlinearities with the same and different scaling factors. The results are validated by numerical simulations using MATLAB. It has more advantage over other synchronization to enhance security of communications function projective synchronization is more unpredictable and moreover it is performed for hyperchaotic system, which makes it more useful.

AUTHORS' CONTRIBUTIONS

Babatunde A. Idowu – developed the statement of the problem, the route to follow as well as necessary tools to utilize, collated all the different sections, their arrangements and also proof read with major emphasis on the introduction and necessary references. The expanded abstract and conclusion was developed with Cornelius O. Ogabi. Kehinde S. Oyeleke and Olasunkanmi I. Olusola – did the analytical calculations as well as the numerical simulations whilst Cornelius O. Ogabi wrote the introduction, re-confirm the analytical calculations and the numerical simulations and re-plotting where necessary.

CONSENT (WHERE EVER APPLICABLE)

Consent form has been approved by all authors.

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